Statics and Dynamics of Global Supply Chain Networks with Environmental Decision-Making

Anna Nagurney, Jose Cruz, Fuminori Toyasaki*
Department of Finance and Operations Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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Abstract: In this paper, we develop static and dynamic models of global supply chains as networks with three tiers of decision-makers: manufacturers, retailers, and consumers associated with the demand markets, who compete within a tier but cooperate between tiers. The decision-makers may be based in the same or in different countries, may transact in different currencies, and are faced with different degrees of environmental concern. Moreover, we allow for electronic transactions in the form of electronic commerce between the decision-makers. The proposed *supernetwork* framework formalizes the modeling and theoretical analysis of such global supply chains, and also enables the dynamic tracking of the evolution of the associated prices and product transactions (as well as the emissions) to the equilibrium state. Moreover, it measures the impacts on the environment associated with the behaviors of the decision-makers. Finally, we propose a discrete-time algorithm which allows for the discretization of the continuous time trajectories and which results in closed form expressions at each iteration.

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1. Introduction

Growing competition and emphasis on efficiency and cost reduction, as well as the satisfaction of consumer demands, have brought new challenges for businesses in the global marketplace. At the same time that businesses and, in particular, supply chains have become increasingly globalized, criticism of globalization has increased, notably, from environmentalists on the basis that free trade may result in the growth of global pollution. In particular, some argue that free trade increases the scale of economic activity and, therefore, of accompanying pollution, and also that it may shift the production of the pollution-intensive goods from countries with strict environmental regulations towards those with lax ones. Others argue that environmental, health, and safety regulations are a form of protectionism. For example, countries may use a laborious and time-consuming regulatory process that is unevenly applied to international investors as a means of controlling access to domestic markets.

Indeed, the increase in environmental concerns is significantly influencing supply chains. Legal requirements and changing consumer preferences increasingly make suppliers, manufacturers, and distributors responsible for their products beyond their sales and delivery locations (cf. Bloemhof-Ruwaard et al. (1995)). For example, recent legislation in the United States as well as abroad and, in particular, in Europe and in Japan, has refocused attention on recycling for the management of wastes and, specifically, that of electronic wastes (see e.g. Appelbaum (2002a) and Nagurney and Toyasaki (2003)). Massachusetts in 2000 banned cathode-ray tubes (CRTs) from landfills whereas Japan in 2001 enacted a law that requires retailers and manufacturers to bear some electronic waste collection and recycling cost of appliances (cf. Appelbaum (2002b, c) and Nagurney and Toyasaki (2005)). In addition, environmental pressure from consumers has, in part, affected the behavior of certain manufacturers so that they attempt to minimize their emissions, produce more environmentally friendly products, and/or establish sound recycling network systems (see, e.g., Bloemhof-Ruwaard et al. (1995), Hill (1997), and Ingram (2002)).

Moreover, according to Fabian (2000), companies are being held accountable not only for their own performance, but also for that of their suppliers, subcontractors, joint venture partners, distribution outlets, and even, ultimately, for the disposal of their products. Indeed, poor environmental performance at any stage of the supply chain process may damage a company's most important asset – its reputation.

On the other hand, innovations in technology and especially the availability of electronic commerce via the Internet in which the physical ordering of goods (and supplies) (and, in some cases, even delivery) is replaced by electronic orders, offers the potential for reducing risks associated with physical transportation due to potential threats and disruptions in supply chains as well as the possible reduction of pollution. Indeed, the introduction of electronic commerce (e-commerce) has unveiled new opportunities for the management of supply chain networks (cf. Nagurney and Dong (2002) and the references therein) and has had an immense effect on the manner in which businesses order goods and have them transported. According to Mullaney et al. (2003) gains from electronic commerce could reach $450 billion a year by 2005, with consumer e-commerce in the United States alone expected to come close to the $108 billion predicted, despite a recession, terrorism, and war.

Many researchers have recently dealt with environmental risks in response to growing environmental concerns (see Batterman (1991), Buck, Hendrix, and Schoorlemmer (1999), Quinn (1999), and Qio, Prato, and McCamley (2001)). Furthermore, the importance of global issues in supply chain management and analysis has been emphasized in several papers (cf. Kogut and Kulatilaka (1994), Cohen and Mallik (1997), Nagurney, Cruz, and Matsypura (2003)). Moreover, earlier surveys on global supply chain analysis indicate that the research interest is growing rapidly (see Erenguc, Simpson, and Vakharia (1999), Cohen and Huchzermeier (1998), and Fabian (2000)). The need to incorporate risk in supply chain decision-making and analysis is well-documented in the literature (see, e.g., Smeltzer and Siferd (1998), Agrawal and Seshadri (2000), Fabian (2000), Johnson (2001), and Zsidisin (2003)). Nevertheless, the topic of supply chain modeling and analysis combined with environmental decision-making is fairly new and novel and, hence, methodological approaches that capture the operational as well as the financial aspects of such decision-making are sorely needed.

Frameworks for risk management in a global supply chain context with a focus on centralized decision-making and optimization have been proposed by Huchzermeier and Cohen (1996), Cohen and Mallik (1997) (see also the references therein) and Fabian (2000). In this paper, in contrast, we build upon the recent work of Nagurney, Cruz, and Matsypura

(2003) in the modeling of global supply chain networks with electronic commerce and that of Nagurney and Toyasaki (2003) who introduced environmental criteria into a decentralized supply chain network.

In particular, in this paper, we develop both static and dynamic global supply chain network models with environmental decision-making handled as a multicriteria decision-making problem. In addition, we build upon our tradition of a network perspective to environmental management as described in the book on environmental networks by Dhanda, Nagurney, and Ramanujam (1999).

The paper is organized as follows. In Section 2, we present the static global supply chain network model with environmental decision-making, derive the optimality conditions for each set of network agents or decision-makers, and provide the finite-dimensional variational ienquality formulation of the governing equilibrium conditions. In Section 3, we propose the projected dynamical system which describes the dynamic adjustment processes associated with the various decision-makers and demonstrate that the set of stationary points of this non-classical dynamical system coincides with the set of solutions of the variational inequality problem (cf. Nagurney and Zhang (1996) and Nagurney (1999)).

In Section 4, we provide qualitative properties of the equilibrium pattern and also provide, under suitable assumptions, existence and uniqueness results for the dynamic price and product transaction trajectories, from which the total emissions generated can also be obtained. In Section 5, we outline the computational procedure, which provides a time-discretization of the dynamic trajectories. We conclude the paper with a summary and suggestions for future research in Section 6.

2. The Global Supply Chain Network Equilibrium Model with Environmental Decision-Making

In this Section, we develop the global supply chain network model and focus on the statics surrounding the equilibrium state. The model assumes that the manufacturers are involved in the production of a homogeneous product and considers L countries, with I manufacturers in each country, and J retailers, which are not country-specific but, rather, can be either physical or virtual, as in the case of electronic commerce. There are K demand markets for the homogeneous product in each country and H currencies in the global economy. We denote a typical country by l or $\hat{l}$, a typical manufacturer by i, and a typical retailer by j. A typical demand market, on the other hand, is denoted by k and a typical currency by h. We assume, for the sake of generality, that each manufacturer can transact directly in an electronic manner via the Internet with the consumers at the demand markets and can also conduct transactions with the retailers either physically or electronically in different currencies. Similarly, we assume that the demand for the product in a country can be associated with a particular currency. We let m refer to a mode of transaction with $m = 1$ denoting a physical transaction and $m = 2$ denoting an electronic transaction via the Internet. In addition, for the sake of flexibility, we assume that the consumers associated with the demand markets can transact with the retailers either physically or electronically. Of course, if either such a transaction is not feasible then one may simply remove that possibility (or, analogously, assign a high associated transaction cost as described below) within the specific application.

The global supply chain *supernetwork* is now described and depicted graphically in Figure 1 (for other supernetwork structures which capture decision-making tradeoffs regarding transportation versus telecommunication networks, see the book by Nagurney and Dong (2002)). The top tier of nodes consists of the manufacturers in the different countries, with manufacturer i in country l being referred to as manufacturer il and associated with node il. There are, hence, IL top-tiered nodes in the network. The middle tier of nodes consists of the retailers (which need not be country-specific) and who act as intermediaries between the manufacturers and the demand markets, with a typical retailer j associated with node j in this (second) tier of nodes in the network. The bottom tier of nodes consists of the demand markets, with a typical demand market k in currency h and country $\hat{l}$ being associated with

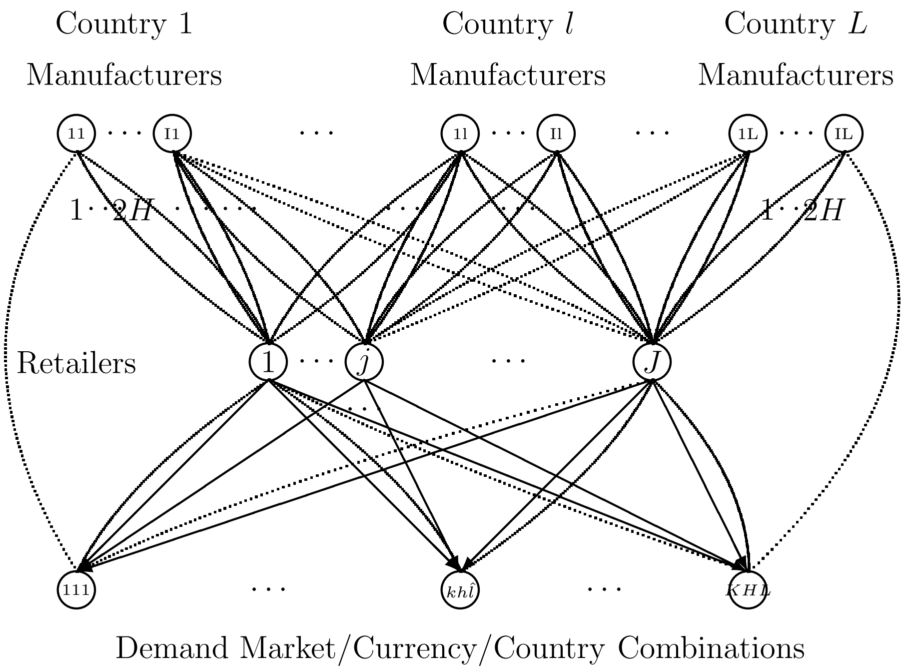

Figure 1: The Structure of the Global Supply Chain Supernetwork

node $kh\hat{l}$ in the bottom tier of nodes. There are, as depicted in Figure 1, J middle (or second) tiered nodes corresponding to the retailers and KHL bottom (or third) tiered nodes in the global supply chain network.

We have identified the nodes in the global supply chain supernetwork and now we turn to the identification of the links joining the nodes in a given tier with those in the subsequent tier. We also associate the product transactions with the appropriate links which correspond to the flows on the links. We assume that each manufacturer i in country l can transact with a given retailer in either of the two modes and in any of the H available currencies, as represented, respectively, by the $2H$ links joining each top tier node with each middle tier node j; $j = 1, \ldots, J$. The flow on the link joining node il with node j and corresponding to transacting via mode m is denoted by q^{il}_{jhm} and represents the nonnegative amount of the product transacted by manufacturer i in country l in currency h through retailer j via mode m. We further group all such transactions for all manufacturers in all countries into the column vector $Q^1 \in R_+^{2ILJH}$.

A manufacturer may also transact directly with the demand markets via the Internet. The flow on the link joining node il with node $kh\hat{l}$ is denoted by $q^{il}_{kh\hat{l}}$ and represents the amount of the product transacted in this manner between the manufacturer and demand market in a given country and currency. We group all such (electronic) transactions for all the manufacturers in all the countries into the column vector $Q^3 \in R_+^{ILKHL}$. For flexibility, we also group the product transactions associated with manufacturer i in country l into the column vector $q^{il} \in R_+^{2JH+KHL}$, and group these vectors for all manufacturers and countries into the vector $q \in R_+^{IL(2JH+KHL)}$.

From each retailer node j; $j = 1, \ldots, J$, we then construct two links to each node khl, with the first such link denoting a physical transaction and the second such link an electronic transaction and with the respective flow on the link being denoted by q^j_{khlm} and corresponding to the amount of the product transacted between retailer j and demand market k in currency h and country l via mode m. The product transactions for all the retailers are then grouped into the column vector $Q^2 \in R_+^{2JKHL}$. Note that if a retailer is virtual, then we expect the transaction to take place electronically, although of course, the product itself may be delivered physically. Nevertheless, for the sake of generality, we allow for two modes of transaction between each manufacturer and retailer pair and each retaler demand market pair.

The notation for the prices is now given. Note that there will be prices associated with each of the tiers of nodes in the global supply chain supernetwork. Let ρ^{il}_{1jhm} denote the price associated with the product in currency h transacted between manufacturer il and retailer j via mode m and group these top tier prices into the column vector $\rho_1 \in R_+^{2ILJH}$. Let $\rho^{il}_{1kh\hat{l}}$, in turn, denote the price associated with manufacturer il and demand market k in currency h and country $\hat{l}$ and group all such prices into the column vector $\rho_{12} \in R_+^{ILKHL}$. Further, let ρ^j_{2khlm}, in turn, denote the price associated with retailer j and demand market k in currency h, country l, and mode m, and group all such prices into the column vector $\rho_2 \in R_+^{2JKHL}$. Also, let $\rho_{3kh\hat{l}}$ denote the price of the product at demand market k in currency h, and country $\hat{l}$, and group all such prices into the column vector $\rho_3 \in R_+^{KHL}$. Finally, we introduce the currency exchange rates: e_h; $h = 1, \ldots, H$, which are the exchange rates of respective currency h relative to the base currency. The exchange rates are exogenous and fixed in the model, whereas all the prices are endogenous.

We now turn to describing the behavior of the various global supply chain network decision-makers represented by the three tiers of nodes in Figure 1. The model is presented, for ease of exposition, for the case of a single homogeneous product. It can also handle multiple products through a replication of the links and added notation. We first focus on the manufacturers. We then turn to the retailers, and, finally, to the consumers at the demand markets.

The Behavior of the Manufacturers

We denote the transaction cost associated with manufacturer il transacting with retailer j for the product in currency h via mode m by c^{il}_{jhm} and assume that:

$$c^{il}_{jhm} = c^{il}_{jhm}(q^{il}_{jhm}), \quad \forall i, l, j, h, m, \tag{1a}$$

that is, this cost can depend upon the volume of this transaction. In addition, we denote the transaction cost associated with manufacturer il transacting with demand market k in country $\hat{l}$ for the product in currency h (via the Internet) by $c^{il}_{kh\hat{l}}$ and assume that:

$$c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}}), \quad \forall i, l, k, h, \hat{l}, \tag{1b}$$

that is, this transaction cost also depends upon the volume of the transaction. These transaction cost functions are assumed to be convex and continuously differentiable. The transaction costs are assumed to be measured in the base currency.

The total transaction costs incurred by manufacturer il are equal to the sum of all of his transaction costs associated with dealing with the distinct retailers and demand markets in the different currencies and countries. His revenue, in turn, is equal to the sum of the price (rate of return plus the rate of appreciation) that the manufacturer can obtain for the product times the total quantity sold of that product. Let now ρ^{il*}_{1jhm} denote the actual price charged by manufacturer il for the product transacted via mode m in currency h to retailer j (and that the retailer is willing to pay) and let $\rho^{il*}_{1kh\hat{l}}$, in turn, denote the price associated with manufacturer il transacting electronically with demand market $kh\hat{l}$. We later discuss how such prices are recovered.

We assume that each manufacturer seeks to maximize his profits. Also, we assume that the amount of the product produced by manufacturer il and denoted by q^{il} must be equal to the amount transacted with the subsequent tiers of nodes, that is,

$$\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il}_{jhm} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} q^{il}_{kh\hat{l}} = q^{il}, \quad \forall i, l. \tag{2}$$

In addition, we assume, as given, a production cost function for manufacturer il and denoted by f^{il}, which depends not only on the manufacturer's output (and transactions) but also on those of the other manufacturers. Hence, we may write (utilizing also (2)) that

$$f^{il} = f^{il}(q) = f^{il}(Q^1, Q^3), \quad \forall i, l. \tag{3}$$

Recall that the vector Q^1 represents all the product transactions between the top tier nodes and the middle tier nodes and the vector Q^3 represents all the product transactions between the top tier nodes and the bottom tier nodes. The function f^{il} is assumed to be strictly convex and continuously differentiable.

We now construct the profit maximization problem facing manufacturer il. In particular, we can express the profit maximization problem facing manufacturer il as:

$$\text{Maximize} \quad \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} (\rho^{il*}_{1jhm} \times e_h) q^{il}_{jhm} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} (\rho^{il*}_{1kh\hat{l}} \times e_h) q^{il}_{kh\hat{l}}$$

$$-\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} c^{il}_{jhm}(q^{il}_{jhm}) - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}}) - f^{il}(Q^1, Q^3), \tag{4}$$

subject to:

$$q^{il}_{jhm} \geq 0, \quad q^{il}_{kh\hat{l}} \geq 0, \quad \forall j, h, k, \hat{l}, m. \tag{5}$$

The first two terms in (4) represent the revenues whereas the subsequent three terms represent the costs faced by the manufacturer.

In addition to the criterion of profit maximization, we assume that each manufacturer is also concerned with environmental decision-making with such decision-making broadly defined as including the risks associated with his transactions. First, we consider the situation that a given manufacturer seeks to minimize the total amount of emissions associated with his production of the product as well as the total amount of emissions generated not only in the ultimate delivery of the product to the next tier of decision-makers (whether retailers or consumers at the demand markets). We assume that the emissions generated by manufacturer il in producing the product are given by the function ϵ^{il}, where

$$\epsilon^{il} = \epsilon^{il}(q^{il}), \quad \forall i, l, \tag{6}$$

whereas the emissions generated in transacting with retailer j for the product via mode m (which are currency independent) are given by a function ϵ^{il}_{jm}, such that

$$\epsilon^{il}_{jm} = \epsilon^{il}_{jm}(\sum_{h=1}^{H} q^{il}_{jhm}), \quad \forall i, l, j, m, \tag{7}$$

and, finally, the emissions generated and associated with the transaction with demand market k in country $\hat{l}$ is represented by a function $\epsilon^{il}_{k\hat{l}}$, where

$$\epsilon^{il}_{k\hat{l}} = \epsilon^{il}_{k\hat{l}}(\sum_{h=1}^{H} q^{il}_{kh\hat{l}}), \quad \forall i, l, k, h, \hat{l}. \tag{8}$$

Note that (8) also does not depend on the currency uitilized for the transaction. Indeed, emissions should not be currency-dependent but, rather, mode-dependent as well as dependent upon the nodes involved in the transaction.

Hence, the second criterion of each manufacturer il and reflecting the minimization of total emissions generated can be expressed mathematically as:

$$\text{Minimize} \quad \epsilon^{il}(q^{il}) + \sum_{j=1}^{J}\sum_{m=1}^{2} \epsilon^{il}_{jm}(\sum_{h=1}^{H} q^{il}_{jhm}) + \sum_{k=1}^{K}\sum_{\hat{l}=1}^{L} \epsilon^{il}_{k\hat{l}}(\sum_{h=1}^{H} q^{il}_{kh\hat{l}}) \tag{9}$$

subject to:

$$q^{il}_{jhm} \geq 0, \quad q^{il}_{kh\hat{l}} \geq 0, \quad \forall j, h, m, k, \hat{l}. \tag{10}$$

From this point on, we consider emission functions of specific form (cf. (6), (7), and (8)) given by

$$\epsilon^{il}(q^{il}) = \eta^{il} q^{il} = \eta^{il}(\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{m=1}^{2} q^{il}_{jhm} + \sum_{k=1}^{K}\sum_{h=1}^{J}\sum_{\hat{l}=1}^{L} q^{il}_{kh\hat{l}}), \quad \forall i, l, \tag{11}$$

$$\epsilon^{il}_{jm}(\sum_{h=1}^{H} q^{il}_{jhm}) = \eta^{il}_{jm} \sum_{h=1}^{H} q^{il}_{jhm}, \quad \forall i, l, j, m, \tag{12}$$

$$\epsilon^{il}_{k\hat{l}}(\sum_{h=1}^{H} q^{il}_{kh\hat{l}}) = \eta^{il}_{k\hat{l}} \sum_{h=1}^{H} q^{il}_{kh\hat{l}}, \quad \forall i, l, k, \hat{l}, \tag{13}$$

where the η^{il}, η^{il}_{jm}, and $\eta^{il}_{k\hat{l}}$ terms are nonnegative and represent the amount of emissions generated per unit of product produced and transacted, respectively. Hence, here we explicitly allow the emissions generated to be distinct according to whether the transaction was conducted electronically or not. Thus, the manufacturer's decision-making problem concerning the emissions generated, in view of (2), (9), and (11)–(13), can be expressed as:

$$\text{Minimize} \quad \sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{m=1}^{2} (\eta^{il} + \eta^{il}_{jm}) q^{il}_{jhm} + \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} (\eta^{il} + \eta^{il}_{k\hat{l}}) q^{il}_{kh\hat{l}}, \tag{14}$$

subject to (10).

We also assume that each manufacturer is concerned with risk minimization and, as noted earlier, here we assume that the risk can also capture environmental risk, with such risk being interpreted broadly. Hence, for the sake of generality, we assume, as given, a risk function r^{il}, for manufacturer il, which is assumed to be continuous and convex, and a function of not only the product transactions associated with the particular manufacturer but also of those of the other manufacturers. Thus, we assume that

$$r^{il} = r^{il}(Q^1, Q^3), \quad \forall i, l. \tag{15}$$

The third criterion of manufacturer il can be expressed as:

$$\text{Minimize} \quad r^{il}(Q^1, Q^3), \tag{16}$$

subject to: $q^{il}_{jhm} \geq 0$, for all j, h, m and $q^{il}_{kh\hat{l}} \geq 0$, for all $k, h, \hat{l}$. The risk function may be distinct for each manufacturer/country combination and can assume whatever form is necessary, provided the above stated assumptions are satisfied.

A Manufacturer's Multicriteria Decision-Making Problem

We now discuss how to construct a value function associated with the criteria. In particular, we assume that manufacturer il assigns nonnegative weights as follows: the weight α^{il} is associated with the emission criterion (14), the weight ω^{il} is associated with the risk criterion (16), with the weight associated with profit maximization (cf. (4)) serving as the numeraire and being set equal to 1. Thus, we can construct a value function for each manufacturer (cf. Fishburn (1970), Chankong and Haimes (1983), Yu (1985), and Keeney and Raiffa (1993)) using a constant additive weight value function. Consequently, the multicriteria decision-making problem for manufacturer il is transformed into:

$$\text{Maximize} \quad \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho_{1jhm}^{il*}\times e_h)q_{jhm}^{il}+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\rho_{1kh\hat{l}}^{il*}\times e_h)q_{kh\hat{l}}^{il}-\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}c_{jhm}^{il}(q_{jhm}^{il})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il})-f^{il}(Q^1,Q^3)$$

$$-\alpha^{il}(\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}(\eta^{il}+\eta_{jm}^{il})q_{jhm}^{il}+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\eta^{il}+\eta_{k\hat{l}}^{il})q_{kh\hat{l}}^{il})-\omega^{il}r^{il}(Q^1,Q^3), \tag{17}$$

subject to the nonnegativity assumption on all the variables.

The Optimality Conditions of the Manufacturers

We assume that the manufacturers compete in a noncooperative fashion following Nash (1950, 1951). Hence, each manufacturer seeks to determine his optimal strategies, that is, the product transactions, given those of the other manufacturers. The optimality conditions of all manufacturers i; $i=1,\dots,I$; in all countries: l; $l=1,\dots,L$, simultaneously, under the above assumptions (see also Gabay and Moulin (1980), Bazaraa, Sherali, and Shetty (1993), Nagurney (1999), Nagurney, Dong, and Zhang (2002)), can be compactly expressed as a variational inequality problem given by: determine $(Q^{1*},Q^{3*})\in\mathcal{K}^1$, satisfying

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q_{jhm}^{il}}+\frac{\partial c_{jhm}^{il}(q_{jhm}^{il*})}{\partial q_{jhm}^{il}}+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q_{jhm}^{il}}+\alpha^{il}(\eta^{il}+\eta_{jm}^{il})\right.$$

$$\left.-\rho_{1jhm}^{il*}\times e_h\right]\times\left[q_{jhm}^{il}-q_{jhm}^{il*}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q_{kh\hat{l}}^{il}}+\frac{\partial c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il*})}{\partial q_{kh\hat{l}}^{il}}+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q_{kh\hat{l}}^{il}}+\alpha^{il}(\eta^{il}+\eta_{k\hat{l}}^{il})\right.$$

$$\left.-\rho_{1kh\hat{l}}^{il*}\times e_h\right]\times\left[q_{kh\hat{l}}^{il}-q_{kh\hat{l}}^{il*}\right]\geq 0, \quad \forall(Q^1,Q^3)\in\mathcal{K}^1, \tag{18}$$

where the feasible set $\mathcal{K}^1 \equiv \{(Q^1, Q^3)|(Q^1, Q^3) \in R_+^{IL(2JH+KHL)}\}$.

The Behavior of the Retailers

The retailers (cf. Figure 1), in turn, are involved in transactions both with the manufacturers in the different countries, as well as with the ultimate consumers associated with the demand markets and represented by the bottom tier of nodes in the network.

A retailer j is faced with what we term a *handling/conversion* cost, which may include, for example, the cost of handling and storing the product plus the cost associated with transacting in the different currencies. We denote such a cost faced by retailer j by c_j and, in the simplest case, we would have that c_j is a function of $\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q_{jhm}^{il}$, that is, the handling/conversion cost of a retailer is a function of how much he has obtained of the product from the various manufacturers in the different countries and what currency the transactions took place in and in what transaction mode. For the sake of generality, however, we allow the function to depend also on the amounts of the product held and transacted by other retailers and, therefore, we may write:

$$c_j = c_j(Q^1), \quad \forall j. \tag{19}$$

The handling cost is measured in the base currency.

The retailers, which can be either physical or virtual, also have associated transaction costs in regards to transacting with the manufacturers, which we assume can be dependent on the type of currency as well as on the manufacturer and country. We denote the transaction cost associated with retailer j transacting with manufacturer il associated with currency h and mode m by $\hat{c}_{jhm}^{il}$ and we assume that it is of the form

$$\hat{c}_{jhm}^{il} = \hat{c}_{jhm}^{il}(q_{jhm}^{il}), \quad \forall i, l, j, h, m, \tag{20a}$$

that is, such a transaction cost depends on the volume of the transaction. In addition, we assume that a retailer j also incurs a transaction cost $c_{kh\hat{l}m}^{j}$ associated with transacting with demand market khl via mode m, where

$$c_{kh\hat{l}m}^{j} = c_{kh\hat{l}m}^{j}(q_{kh\hat{l}m}^{j}), \quad \forall j, k, h, \hat{l}, m. \tag{20b}$$

Hence, the transaction costs given in (20b) can vary according to the retailer/currency/-country combination and are a function of the volume of the product transacted. We assume that the cost functions (19) – (20) are convex and continuously differentiable and are measured in the base currency.

The actual price charged for the product by retailer j is denoted by $\rho^{j*}_{2kh\hat{l}m}$ and is associated with transacting with consumers at demand market k in currency h and country l via mode m. Subsequently, we discuss how such prices are arrived at. We assume that the retailers are also profit-maximizers, with the criterion of profit maximization for retailer j given by:

$$\text{Maximize} \quad \sum_{k=1}^{k}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}(\rho^{j*}_{2kh\hat{l}m} \times e_h)q^{j}_{kh\hat{l}m} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{c}^{il}_{jhm}(q^{il}_{jhm})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{m=1}^{2}\sum_{\hat{l}=1}^{L}c^{j}_{kh\hat{l}m}(q^{j}_{kh\hat{l}m}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho^{il*}_{1jhm} \times e_h)q^{il}_{jhm} \tag{21}$$

subject to the nonnegativity constraints:

$$q^{il}_{jhm} \geq 0, \quad q^{j}_{kh\hat{l}m} \geq 0, \quad \forall i, \hat{l}, h, m, \tag{22}$$

and

$$\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q^{j}_{kh\hat{l}m} \leq \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il}_{jhm}. \tag{23}$$

Objective function (21) expresses that the difference between the revenues (given by the first term) minus the handling cost, the two sets of transaction costs, and the payout to the manufacturers (given by the fifth term) should be maximized. The objective function in (21) is concave in its variables under the above posed assumptions. Constraint (23) guarantees that a retailer does not transact more of the product with the demand markets than he has in his possession.

We now turn to describing the criteria associated with a retailer's environmental decision-making similar to that developed above for a given manufacturer. Hence, we allow the retailers to also be faced with multiple criteria.

In particular, we assume that retailer j seeks to minimize the emissions associated with his transactions with the manufacturers, that is, he also is faced with the following problem:

$$\text{Minimize} \quad \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(\eta^{il}+\eta^{il}_{jm})q^{il}_{jhm} \tag{24}$$

subject to: $q^{il}_{jhm} \geq 0, \quad \forall i, l, h, m.$

Note that we do not consider a retailer's decision-making concerning emissions generated to involve the demand markets (since, in a sense, this may be viewed as discriminatory). Below we describe how environmental decision-making is captured at the demand market level.

Moreover, each retailer seeks to also minimize the risk associated with obtaining the product from the manufacturers and transacting with the various demand markets, which we assume to also include a general form of environmental risk.

Hence, each retailer j is faced with his own individual risk denoted by r^j with the function being assumed to be continuous and convex and dependent on the transactions to and from all the retailers, that is,

$$r^j = r^j(Q^1, Q^2), \quad \forall j. \tag{25}$$

The third criterion or retailer j can be expressed as:

$$\text{Minimize} \quad r^j(Q^1, Q^2) \tag{26}$$

subject to: $q^{il}_{jhm} \geq 0$, for all i, l, h, m and $q^{j}_{kh\hat{l}} \geq 0$, for all $k, h, \hat{l}$.

A Retailer's Multicriteria Decision-Making Problem

We now demonstrate (akin to the above construction for a given manufacturer) how the multiple criteria faced by a retailer can be transformed into a single optimization problem using, again, the concept of a value function.

In particular, we assume that retailer j associates a nonnegative weight β_j with the emission generation criterion (24), a weight ϑ_j with the risk criterion (26), and a weight equal to 1 with profit maximization (cf. (21)) (see also the discussion concerning the manufacturers above), yielding the following multicriteria decision-making problem:

$$\text{Maximize} \quad \sum_{k=1}^{k}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}(\rho_{2kh\hat{l}m}^{j*}\times e_h)q_{kh\hat{l}m}^{j} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{c}_{jhm}^{il}(q_{jhm}^{il})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{m=1}^{2}\sum_{\hat{l}=1}^{L}c_{kh\hat{l}m}^{j}(q_{kh\hat{l}m}^{j}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho_{1jhm}^{il*}\times e_h)q_{jhm}^{il}$$

$$-\beta_j(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(\eta^{il}+\eta_{jm}^{il})q_{jhm}^{il}) - \vartheta_j r^j(Q^1,Q^2) \tag{27}$$

subject to: the nonnegativity assumption on the variables and (23).

Optimality Conditions of the Retailers

Here we assume that the retailers can also compete in a noncooperative manner with the governing optimality/equilibrium concept being that of Nash. The optimality conditions for all retailers, simultaneously, under the above stated assumptions, can, hence, be expressed as the variational inequality problem: determine $(Q^{1*}, Q^{2*}, \gamma^*) \in \mathcal{K}^2$, such that

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{jhm}^{il}} + \vartheta_j\frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q_{jhm}^{il}} + \beta_j(\eta^{il}+\eta_{jm}^{il}) + \rho_{1jhm}^{il*}\times e_h\right.$$

$$\left.+\frac{\partial \hat{c}_{jhm}^{il}(q_{jhm}^{il*})}{\partial q_{jhm}^{il}} - \gamma_j^*\right]\times\left[q_{jhm}^{il}-q_{jhm}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial c_{kh\hat{l}m}^{j}(q_{kh\hat{l}m}^{j*})}{\partial q_{kh\hat{l}m}^{j}} + \vartheta_j\frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q_{kh\hat{l}m}^{j}} - \rho_{2kh\hat{l}m}^{j*}\times e_h + \gamma_j^*\right]\times\left[q_{kh\hat{l}m}^{j}-q_{kh\hat{l}m}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}q_{jhm}^{il*} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}q_{kh\hat{l}m}^{j*}\right]\times\left[\gamma_j-\gamma_j^*\right]\geq 0, \quad \forall (Q^1,Q^2,\gamma)\in\mathcal{K}^2, \tag{28}$$

where the feasible set $\mathcal{K}^2 \equiv \{(Q^1, Q^2, \gamma) \in R_+^{2ILJH+2JKHL+J}\}$.

Here γ_j denotes the Lagrange multiplier associated with constraint (23) (see Bazaraa, Sherali, and Shetty (1993)), and γ is the J-dimensional column vector of Lagrange multipliers of all the retailers with γ^* denoting the vector of optimal multipliers. Note that γ_j^* serves as the market clearing price for the product at retailer j (as can be seen from the last term in (28)). In particular, its value is positive if the product transactions from the retailer to all the demand markets in the countries and in the various currencies is precisely equal to the product transactions to the retailer from all the manufacturers in all the countries transacted in the different currencies (and modes).

The Equilibrium Conditions at the Demand Markets

We now describe the consumers located at the demand markets. The consumers take into account in making their consumption decisions not only the price charged for product by the manufacturer and by the retailers but also their transaction costs associated with obtaining the product. We also describe how their environmental decision-making is captured.

We let $\hat{c}^j_{kh\hat{l}m}$ denote the transaction cost associated with consumers obtaining the product at demand market k in currency h and in country $\hat{l}$ via mode m from retailer j and recall that $q^j_{kh\hat{l}m}$ is the amount of the product transacted thus. We assume that the transaction cost function is continuous and of the general form:

$$\hat{c}^j_{kh\hat{l}m} = \hat{c}^j_{kh\hat{l}m}(Q^2), \quad \forall j, k, h, \hat{l}, m. \tag{29a}$$

Furthermore, let $\hat{c}^{il}_{kh\hat{l}}$ denote the transaction cost associated with consumers obtaining the product at demand market k in currency h and in country $\hat{l}$ transacted electronically from manufacturer il, where we assume that the transaction cost is continuous and of the general form:

$$\hat{c}^{il}_{kh\hat{l}} = \hat{c}^{il}_{kh\hat{l}}(Q^3), \quad \forall i, l, k, h, \hat{l}. \tag{29b}$$

Hence, the transaction cost associated with transacting directly with manufacturers is of a form of the same level of generality as the transaction costs associated with transacting with the retailers. Note that the above functional forms can capture congestion on the networks. Indeed, we allow for the transaction cost (from the perspective of consumers) to depend not only upon the flow of the product from a manufacturer or from the retailer in the currency to the country (and mode) but also on the other product transactions in the other currencies and between other manufacturers and/or retailers and demand markets. The transaction cost functions above are assumed to be measured in the base currency.

Denote now the demand for the product at demand market k in currency h in country $\hat{l}$

by $d_{kh\hat{l}}$ and assume, as given, the continuous demand functions:

$$d_{kh\hat{l}} = d_{kh\hat{l}}(\rho_3), \quad \forall k, h, \hat{l}. \tag{30}$$

Thus, according to (30), the demand for the product at a demand market in a currency and country depends, in general, not only on the price of the product at that demand market (and currency and country) but also on the prices of the product at the other demand markets (and in other countries and currencies). Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

The consumers take the price charged by the retailer, which was denoted by $\rho^{j*}_{2kh\hat{l}m}$ for retailer j, demand market k, currency h, and country $\hat{l}$ transacted via mode m, the price charged by manufacturer il, which was denoted by $\rho^{il*}_{1kh\hat{l}}$, and the rate of appreciation in the currency, plus the transaction costs, in making their consumption decisions. In addition, we assume that the consumers are also multicriteria decision-makers and weight the emissions associated with their transactions accordingly.

The Multicriteria Equilibrium Conditions for the Demand Markets

Let $\eta^j_{k\hat{l}m}$ denote the amount of emissions generated per unit of product transacted between retailer j and demand market k in country $\hat{l}$ via mode m and assume that this term is nonnegative for each $k, \hat{l}, m, j$. We assume that consumers at a demand market perceive the emissions generated through their transactions (and purchases) in an individual fashion with the nonnegative weight $\delta_{kh\hat{l}}$ associated with the total emissions generated through consumer transactions at demand market $kh\hat{l}$. This term may also be viewed as a monetary conversion factor associated with the per unit emissions generated. See also Nagurney and Toyasaki (2003).

The equilibrium conditions for the consumers at demand market $kh\hat{l}$, thus, take the form: for all retailers: $j = 1, \ldots, J$ and all modes m; $m = 1, 2$:

$$\rho^{j*}_{2kh\hat{l}m} \times e_h + \hat{c}^j_{kh\hat{l}m}(Q^{2*}) + \delta_{kh\hat{l}}\eta^j_{k\hat{l}m} \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{j*}_{kh\hat{l}m} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{j*}_{kh\hat{l}m} = 0, \end{cases} \tag{31}$$

and for all manufacturers il; $i = 1, \ldots, I$ and $l = 1, \ldots, L$:

$$\rho^{il*}_{1kh\hat{l}} \times e_h + \hat{c}^{il}_{kh\hat{l}}(Q^{3*}) + \delta_{kh\hat{l}}(\eta^{il} + \eta^{il}_{k\hat{l}}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{il*}_{kh\hat{l}} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{il*}_{kh\hat{l}} = 0. \end{cases} \tag{32}$$

In addition, we must have that

$$d_{kh\hat{l}}(\rho^*_3) \begin{cases} = \sum_{j=1}^{J} \sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} > 0 \\ \leq \sum_{j=1}^{J} \sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} = 0. \end{cases} \tag{33}$$

Condition (31) states that consumers at demand market $kh\hat{l}$ will purchase the product from retailer j transacted via mode m, if the price charged by the retailer for the product and the appreciation rate for the currency plus the transaction cost (from the perspective of the consumer) and the weighted emission generation term does not exceed the price that the consumers are willing to pay for the product in that currency and country, i.e., $\rho^*_{3kh\hat{l}}$. Note that, according to (31), if the transaction costs are identically equal to zero, as is the weighted emission generation term, then the price faced by the consumers at a demand market is the price charged by the retailer for the particular product and currency and mode in the country plus the rate of appreciation in the currency. Condition (32) state the analogue, but for the case of electronic transactions with the manufacturers.

Condition (33), on the other hand, states that, if the price the consumers are willing to pay for the product at a demand market/currency/country is positive, then the quantity of the product transacted at the demand market/currency/country is precisely equal to the demand.

In equilibrium, conditions (31), (32), and (33) will have to hold for all demand markets in all countries, currencies, and modes. Hence, these equilibrium conditions can be expressed also as a variational inequality analogous to those in (18) and (28) and given by: determine $(Q^{2*}, Q^{3*}, \rho^*_3) \in R^{(IL+2J+1)KHL}_+$, such that

$$\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{\hat{L}} \sum_{m=1}^{2} \left[\rho^{j*}_{2kh\hat{l}m} \times e_h + \hat{c}^{j}_{kh\hat{l}m}(Q^{2*}) + \delta_{kh\hat{l}}\eta^{j}_{k\hat{l}m} - \rho^*_{3kh\hat{l}}\right] \times \left[q^{j}_{kh\hat{l}m} - q^{j*}_{kh\hat{l}m}\right]$$

$$
+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\rho_{1kh\hat{l}}^{il*}\times e_h+\hat{c}_{kh\hat{l}}^{il}(Q^{3*})+\delta_{kh\hat{l}}(\eta^{il}+\eta_{k\hat{l}}^{il})-\rho_{3kh\hat{l}}^{*}\right]\times\left[q_{kh\hat{l}}^{il}-q_{kh\hat{l}}^{il*}\right]
$$

$$
+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}q_{kh\hat{l}m}^{j*}+\sum_{i=1}^{I}\sum_{l=1}^{L}q_{kh\hat{l}}^{il*}-d_{kh\hat{l}}(\rho_3^*)\right]\times\left[\rho_{3kh\hat{l}}-\rho_{3kh\hat{l}}^{*}\right]\geq 0,
$$

$$
\forall(Q^2,Q^3,\rho_3)\in R_+^{(IL+2J+1)KHL}. \tag{34}
$$

The Equilibrium Conditions for the Global Supply Chain Network

In equilibrium, the product transactions between the manufacturers in the different countries with the retailers must coincide with those that the retailers actually accept. In addition, the amounts of the product that are obtained by the consumers in the different countries and currencies must be equal to the amounts that the retailers and the manufacturers actually provide. Hence, although there may be competition between decision-makers at the same tier of nodes of the supply chain supernetwork there must be, in a sense, cooperation between decision-makers associated with distinct tiers of nodes. Thus, in equilibrium, the prices and product transactions must satisfy the sum of the optimality conditions (18) and (28) and (34). We make these relationships rigorous through the subsequent definition and variational inequality derivation below.

Definition 1: Global Supply Chain Network Equilibrium

The equilibrium state of the supply chain supernetwork is one where the product transactions between the tiers of the network coincide and the product transactions and prices satisfy the sum of conditions (18), (28), and (34).

The equilibrium state is equivalent to the following:

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the global supply chain supernetwork according to Definition 1 are equivalent to the solution of the variational inequality given by: determine

$(Q^{1*}, Q^{2*}, Q^{3*}, \gamma^*, \rho_3^*) \in \mathcal{K}$, *satisfying:*

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q_{jhm}^{il}}+\frac{\partial c_{jhm}^{il}(q_{jhm}^{il*})}{\partial q_{jhm}^{il}}+\frac{\partial c_j(Q^{1*})}{\partial q_{jhm}^{il}}+\frac{\partial \hat{c}_{jhm}^{il}(q_{jhm}^{il*})}{\partial q_{jhm}^{il}}\right.$$

$$\left.+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q_{jhm}^{il}}+\vartheta_j\frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q_{jhm}^{il}}+(\alpha^{il}+\beta_j)(\eta^{il}+\eta_{jm}^{il})-\gamma_j^*\right]\times\left[q_{jhm}^{il}-q_{jhm}^{il*}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q_{kh\hat{l}}^{il}}+\frac{\partial c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il*})}{\partial q_{kh\hat{l}}^{il}}+\hat{c}_{kh\hat{l}}^{il}(Q^{3*})\right.$$

$$\left.+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q_{kh\hat{l}}^{il}}+(\alpha^{il}+\delta_{kh\hat{l}})(\eta^{il}+\eta_{k\hat{l}}^{il})-\rho_{3kh\hat{l}}^*\right]\times\left[q_{kh\hat{l}}^{il}-q_{kh\hat{l}}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial c_{khl}^{j}(q_{kh\hat{l}m}^{j*})}{\partial q_{kh\hat{l}m}{}^{j}}+\hat{c}_{kh\hat{l}m}^{j}(Q^{2*})+\vartheta_j\frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q_{kh\hat{l}m}^{j}}\right.$$

$$\left.+\delta_{kh\hat{l}}\eta_{k\hat{l}m}^{j}+\gamma_j^*-\rho_{3kh\hat{l}}^*\right]\times\left[q_{kh\hat{l}m}^{j}-q_{kh\hat{l}m}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}q_{jhm}^{il*}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}q_{kh\hat{l}m}^{j*}\right]\times\left[\gamma_j-\gamma_j^*\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}q_{kh\hat{l}m}^{j*}+\sum_{i=1}^{I}\sum_{l=1}^{L}q_{kh\hat{l}}^{il*}-d_{kh\hat{l}}(\rho_3^*)\right]\times\left[\rho_{3kh\hat{l}}-\rho_{3kh\hat{l}}^*\right]\geq 0,$$

$$\forall(Q^1,Q^2,Q^3,\gamma,\rho_3)\in\mathcal{K}, \qquad (35)$$

where $\mathcal{K}\equiv\{\mathcal{K}^1\times\mathcal{K}^2\times\mathcal{K}^3\}$, *where* $\mathcal{K}^3\equiv\{\rho_3|\rho_3\in R_+^{KHL}\}$.

Proof: We first establish that the equilibrium conditions imply variational inequality (35). Indeed, summation of inequalities (18), (28), and (34), after algebraic simplifications, yields variational inequality (35).

We now establish the converse, that is, that a solution to variational inequality (35) satisfies the sum of conditions (18), (28), and (34), and is, hence, an equilibrium.

To inequality (35), add the term: $-\rho_{1jhm}^{il*}\times e_h+\rho_{1jhm}^{il*}\times e_h$ to the term in the first set of brackets (preceding the first multiplication sign). Similarly, add the terms: $-\rho_{1kh\hat{l}}^{il*}\times e_h+\rho_{1kh\hat{l}}^{il*}\times e_h$ to the term in brackets preceding the second multiplication sign and $-\rho_{2kh\hat{l}m}^{j*}\times$

$e_h + \rho^{j*}_{2kh\hat{l}m} \times e_h$ to the term in brackets preceding the third multiplication sign in (35). The addition of such terms does not change (35) since the value of these terms is zero and yields:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}} + \frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}}\right.$$
$$+(\alpha^{il}+\beta_j)(\eta^{il}+\eta^{il}_{jm}) + \omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \vartheta_j\frac{\partial r^{j}(Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}} - \gamma^*_j$$
$$\left.-\rho^{il*}_{1jhm}\times e_h + \rho^{il*}_{1jhm}\times e_h\right]\times\left[q^{il}_{jhm}-q^{il*}_{jhm}\right]$$
$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} + \hat{c}^{il}_{kh\hat{l}}(Q^{3*}) + (\alpha^{il}+\delta_{kh\hat{l}})(\eta^{il}+\eta^{il}_{k\hat{l}})\right.$$
$$\left.+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}} - \rho^*_{3kh\hat{l}} - \rho^{il*}_{1kh\hat{l}}\times e_h + \rho^{il*}_{1kh\hat{l}}\times e_h\right]\times\left[q^{il}_{kh\hat{l}}-q^{il*}_{kh\hat{l}}\right]$$
$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial c^{j}_{khl}(q^{j*}_{kh\hat{l}m})}{\partial q^{j}_{kh\hat{l}m}} + \gamma^*_j + \hat{c}^{j}_{kh\hat{l}m}(Q^{2*}) + \vartheta_j\frac{\partial r^{j}(Q^{1*},Q^{2*})}{\partial q^{j}_{kh\hat{l}m}} + \delta_{kh\hat{l}}\eta^{j}_{k\hat{l}m}\right.$$
$$\left.-\rho^*_{3kh\hat{l}} - \rho^{j*}_{2kh\hat{l}m}\times e_h + \rho^{j*}_{2kh\hat{l}m} + e^*_h\right]\times\left[q^{j}_{kh\hat{l}m}-q^{j*}_{kh\hat{l}m}\right]$$
$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}q^{il*}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{\hat{L}}\sum_{m=1}^{2}q^{j*}_{kh\hat{l}m}\right]\times\left[\gamma_j-\gamma^*_j\right]$$
$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I}\sum_{l=1}^{L}q^{il*}_{kh\hat{l}} - d_{kh\hat{l}}(\rho^*_3)\right]\times\left[\rho_{3kh\hat{l}}-\rho^*_{3kh\hat{l}}\right]\geq 0,$$
$$\forall (Q^1,Q^2,Q^3,\gamma,\rho_3)\in\mathcal{K}, \tag{36}$$

which, in turn, can be rewritten as:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}} + \omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \alpha^{il}(\eta^{il}+\eta^{il}_{jm})\right.$$
$$\left.-\rho^{il*}_{1jhm}\times e_h\right]\times\left[q^{il}_{jhm}-q^{il*}_{jhm}\right]$$
$$+\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jhm}} + \rho^{il*}_{1jhm}\times e_h + \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}} + \vartheta_j\frac{\partial r^{j}(Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}}\right.$$
$$\left.+\beta_j(\eta^{il}+\eta^{il}_{jm}) - \gamma^*_j\right]\times\left[q^{il}_{jhm}-q^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}}+\frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}}+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}}+\alpha^{il}(\eta^{il}+\eta^{il}_{k\hat{l}})\right.$$

$$\left.-\rho^{il*}_{1kh\hat{l}}\times e_h\right]\times\left[q^{il}_{kh\hat{l}}-q^{il*}_{kh\hat{l}}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\rho^{il*}_{1kh\hat{l}}\times e_h+\hat{c}^{il}_{kh\hat{l}}(Q^{3*})+\delta_{kh\hat{l}}(\eta^{il}+\eta^{il}_{k\hat{l}})-\rho^{*}_{3kh\hat{l}}\right]\times\left[q^{il}_{kh\hat{l}}-q^{il*}_{kh\hat{l}}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial c^{j}_{khl}(q^{j*}_{kh\hat{l}m})}{\partial q^{j}_{kh\hat{l}m}}+\vartheta_j\frac{\partial r^{j}(Q^{1*},Q^{2*})}{\partial q^{j}_{kh\hat{l}m}}-\rho^{j*}_{2kh\hat{l}m}\times e_h+\gamma^{*}_{j}\right]\times\left[q^{j}_{kh\hat{l}m}-q^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\rho^{j*}_{2kh\hat{l}m}\times e_h+\hat{c}^{j}_{kh\hat{l}m}(Q^{2*})+\delta_{kh\hat{l}}\eta^{j}_{k\hat{l}m}-\rho^{*}_{3kh\hat{l}}\right]\times\left[q^{j}_{kh\hat{l}m}-q^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}q^{il*}_{jhm}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}q^{j*}_{kh\hat{l}m}\right]\times\left[\gamma_j-\gamma^{*}_{j}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}q^{j*}_{kh\hat{l}m}+\sum_{i=1}^{I}\sum_{l=1}^{L}q^{il*}_{kh\hat{l}}-d_{kh\hat{l}}(\rho^{*}_{3})\right]\times\left[\rho_{3kh\hat{l}}-\rho^{*}_{3kh\hat{l}}\right]\geq 0. \tag{37}$$

But inequality (37) is equivalent to the sum of conditions (18), (28), and (34), and, hence, the product and price pattern is an equilibrium according to Definition 1. □

We now put variational inequality (35) into standard form which will be utilized in the subsequent sections. For additional background on variational inequalities and their applications, see the book by Nagurney (1999). For other applications of supernetworks along with the variational inequality formulations of the governing equilibrium conditions see the book by Nagurney and Dong (2002).

In particular, we have that variational inequality (35) can be expressed as:

$$\langle F(X^*)^T, X-X^*\rangle \geq 0, \quad \forall X\in\mathcal{K}, \tag{38}$$

where $X\equiv(Q^1,Q^2,Q^3,\gamma,\rho_3)$ and $F(X)\equiv(F_{iljhm},F_{ilkh\hat{l}},F_{jkh\hat{l}m},F_j,F_{kh\hat{l}})_{i=1,\ldots,I;\hat{l}=l=1,\ldots,L;j=1,\ldots,J;}$ $_{h=1,\ldots,H};\ _{m=1,2}$, and the specific components of F are given by the functional terms preceding the multiplication signs in (35), respectively. The term $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space.

We now describe how to recover the prices associated with the first two tiers of nodes in the global supply chain network. Clearly, the components of the vector ρ_3^* are obtained directly from the solution of variational inequality (35). In order to recover the second tier prices associated with the retailers and the appreciation rates one can (after solving variational inequality (35) for the particular numerical problem) *either* (cf. (31)) set $\rho^{j*}_{2kh\hat{l}m} \times e_h = \rho^*_{3kh\hat{l}} - \hat{c}^j_{kh\hat{l}m}(Q^{2*}) - \delta_{kh\hat{l}}\eta^j_{k\hat{l}m}$, for any $j, k, h, \hat{l}, m$ such that $q^{j*}_{kh\hat{l}m} > 0$, *or* (cf. (28)) for any $q^{j*}_{kh\hat{l}m} > 0$, set $\rho^{j*}_{2kh\hat{l}m} \times e_h = \frac{\partial c^j_{kh\hat{l}m}(q^{j*}_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} + \vartheta_j \frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q^j_{kh\hat{l}m}} + \gamma^*_j$.

Similarly, from (28) we can infer that the top tier prices comprising the vector ρ_1^* can be recovered (once the variational inequality (35) is solved with particular data) thus: for any i, l, j, h, m, such that $q^{il*}_{jhm} > 0$, set (cf. (18)) $\rho^{il*}_{1jhm} \times e_h = \frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}} + \omega^{il} \frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \alpha^{il}(\eta^{il} + \eta^{il}_{jm})$.

Similarly, (cf. (32)) set $\rho^{il*}_{1kh\hat{l}} \times e_h = \rho^*_{3kh\hat{l}} - \hat{c}^{il}_{kh\hat{l}}(Q^{3*}) - \delta_{kh\hat{l}}(\eta^{il} + \eta^{il}_{k\hat{l}})$, for any $i, l, k, h, \hat{l}$ such that $q^{il*}_{kh\hat{l}} > 0$, *or* (cf. (18)) for any $q^{il*}_{kh\hat{l}} > 0$, set $\rho^{il*}_{1kh\hat{l}} \times e_h = \frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} + \omega^{il} \frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}} + \alpha^{il}(\eta^{il} + \eta^{il}_{k\hat{l}})$.

With the pricing mechanism described above it is straightforward to verify that a solution of variational inequality (35) also satisfies the optimality conditions (18) and (28) as well as the equilibrium conditions (31) – (33) (see also (34)).

3. The Dynamic Global Supply Chain Network Model

In this Section, we turn to the development of a dynamic global supply chain network model whose set of stationary points coincides with the set of solutions of the variational inequality problem (35) governing the static global supply chain network equilibrium model developed in Section 2. In particular, we propose a dynamical system, which is non-classical, and termed a *projected dynamical system* (cf. Nagurney and Zhang (1996), that governs the behavior of the global supply chain supernetwork presented in Section 2. The proposed dynamic adjustment processes describe the disequilibrium dynamics as the various global supply chain decision-makers adjust their product transactions between the tiers and the prices associated with the different tiers adjust as well.

Demand Market Price Dynamics

The rate of change of the price $\rho_{3kh\hat{l}}$, denoted by $\dot{\rho}_{3kh\hat{l}}$, is assumed to be equal to the difference between the demand for the product at the demand market and currency and country and the amount of the product actually available there. Moreover, if the demand for the product at the demand market (and currency and country) at an instant in time exceeds the amount available from the various retailers and manufacturers, then the price will increase; if the amount available exceeds the demand at the price, then the price will decrease. We also have to make sure that the prices do not become negative. Therefore, the dynamics of the prices $\rho_{3kh\hat{l}}$, $\forall k, h, \hat{l}$, can be expressed in the following way:

$$\dot{\rho}_{3kh\hat{l}} = \begin{cases} d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J}\sum_{m=1}^{2} q^j_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il}_{kh\hat{l}}, & \text{if} \quad \rho_{3kh\hat{l}} > 0 \\ \max\{0, d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J}\sum_{m=1}^{2} q^j_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il}_{kh\hat{l}}\} & \text{if} \quad \rho_{3kh\hat{l}} = 0. \end{cases} \quad (39)$$

The Dynamics of the Product Transactions between the Retailers and the Demand Markets

We assume that the rate of change of the product transaction between retailer j and demand market k, country $\hat{l}$, and transacting in currency h via mode m and denoted by $\dot{q}^j_{kh\hat{l}m}$ is equal to the difference between the price consumers at this particular demand market/country/currency combination are willing to pay for the product minus the price charged by the retailer and the various transaction costs and weighted marginal risk and weighted emissions generated. Here we also have to guarantee that the product transactions will not become negative. Thus, the rate of change of the product transactions between a retailer and a demand market in a country and currency via a mode can be written as: $\forall j, k, h, \hat{l}, m$:

$$\dot{q}^j_{kh\hat{l}m} = \begin{cases} \rho_{3kh\hat{l}} - \frac{\partial c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^j_{kh\hat{l}m}} - \hat{c}^j_{kh\hat{l}m}(Q^2) - \delta_{kh\hat{l}}\eta^j_{k\hat{l}m} - \gamma_j, & \text{if} \quad q^j_{kh\hat{l}m} > 0 \\ \max\{0, \rho_{3kh\hat{l}} - \frac{\partial c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^j_{kh\hat{l}m}} - \hat{c}^j_{kh\hat{l}m}(Q^2) - \delta_{kh\hat{l}}\eta^j_{k\hat{l}m} - \gamma_j\}, & \text{if} \quad q^j_{kh\hat{l}m} = 0. \end{cases} \tag{40}$$

The Dynamics of the Prices at the Retailers

The prices at the retailers, whether they are physical or virtual, must reflect supply and demand conditions as well. In particular, we let $\dot{\gamma}_j$ denote the rate of change in the market clearing price associated with retailer j and we propose the following dynamic adjustment for retailer j:

$$\dot{\gamma}_j = \begin{cases} \sum_{k=1}^K \sum_{h=1}^H \sum_{\hat{l}=1}^L \sum_{m=1}^2 q^j_{kh\hat{l}m} - \sum_{i=1}^I \sum_{l=1}^L \sum_{h=1}^H \sum_{m=1}^2 q^{il}_{jhm}, & \text{if} \quad \gamma_j > 0 \\ \max\{0, \sum_{k=1}^K \sum_{h=1}^H \sum_{\hat{l}=1}^L \sum_{m=1}^2 q^j_{kh\hat{l}m} - \sum_{i=1}^I \sum_{l=1}^L \sum_{h=1}^H \sum_{m=1}^2 q^{il}_{jhm}\}, & \text{if} \quad \gamma_j = 0. \end{cases} \tag{41}$$

Hence, if there is excess supply of the product at a retailer, then the price will decrease at that retailer; if there is excess demand then the price will increase. Here we also guarantee that these prices do not become negative.

The Dynamics of the Product Transactions between Manufacturers and Retailers

The dynamics of the product transactions between manufacturers in the countries and the retailers in the different currencies and modes are now described. Note that in order for a transaction between nodes in these two tiers to take place there must be agreement between the pair of decision-makers. Towards that end, we let $\dot{q}^{il}_{jhm}$ denote the rate of change of the product transaction between manufacturer il and retailer j transacted via mode m and in currency h and we have that for every i, l, j, h, m:

$$
\dot{q}^{il}_{jhm} = \begin{cases} \gamma_j - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} \\ -\omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^{il}_{jhm}} - (\alpha^{il} + \beta_j)(\eta^{il} + \eta^{il}_{jm}), & \text{if} \quad q^{il}_{jhm} > 0 \\ \max\{0, \gamma_j - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} \\ -\omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^{il}_{jhm}} - (\alpha^{il} + \beta_j)(\eta^{il} + \eta^{il}_{jm})\}, & \text{if} \quad q^{il}_{jhm} = 0. \end{cases} \tag{42}
$$

Hence, the transaction between a manufacturer in a country and a retailer via a mode and in a currency will increase if the price that the retailer is willing to pay the manufacturer exceeds the various marginal costs plus the weighted marginal risks and emissions generated. Moreover, we quarantee that such a transaction mever becomes negative.

The Dynamics of the Product Transactions between Manufacturers and Demand Markets

The rate of change of the product transactions between a manufacturer in a country and demand market/currency/country pair is assumed to be equal to the price the consumers are willing to pay minus the various costs, including marginal ones, that the manufacturer incurs when transacting with the demand market in a country and currency and the weighted emissions generated and the weighted marginal risk. We denote this rate of change by $\dot{q}^{il}_{kh\hat{l}}$, and, mathematically, express it in the following way, $\forall i, l, k, h, \hat{l}$:

$$
\dot{q}^{il}_{kh\hat{l}} = \begin{cases} \rho_{3kh\hat{l}} - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} - \hat{c}^{il}_{kh\hat{l}}(Q^3) \\ -\omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - (\alpha^{il} + \delta_{kh\hat{l}})(\eta^{il} + \eta^{il}_{k\hat{l}}), & \text{if} \quad q^{il}_{kh\hat{l}} > 0 \\ \max\{0, \rho_{3kh\hat{l}} - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} - \hat{c}^{il}_{kh\hat{l}}(Q^3) \\ -\omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - (\alpha^{il} + \delta_{kh\hat{l}})(\eta^{il} + \eta^{il}_{k\hat{l}})\}, & \text{if} \quad q^{il}_{kh\hat{l}} = 0. \end{cases} \tag{43}
$$

Note that (43) guarantees that the volume of product transacted will not take on a negative value.

The Projected Dynamical System

Consider now a dynamical system in which the product transactions between manufacturers in the countries and the retailers evolve according to (42); the product transactions between manufacturers and demand markets in the various countries and associated with different currencies evolve according to (43); the product transactions between retailers and the demand market/country/currency combinations evolve according to (40); the prices at the retailers evolve according to (41), and the prices at the demand markets evolve according to (39). Let X and $F(X)$ be as defined following (35) and recall also the feasible set $\mathcal{K}$ as defined following (35). Then the dynamic model described by (39) – (43) can be rewritten as a *projected dynamical system* (see Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{44}$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X and $X_0 = (Q^{10}, Q^{20}, Q^{30}, \gamma^0, \rho_3^0)$ is the initial point corresponding to the initial product flow and price pattern. Note that since the feasible set $\mathcal{K}$ is simply the nonnegative orthant the projection operation takes on a very simple form as revealed through (39) – (43).

The trajectory of (44) describes the dynamic evolution of and the dynamic interactions among the product transactions and prices. The dynamical system (44) is non-classical since it has a discontinuous right-hand side due to the projection operation. Such dynamical systems were introduced by Dupuis and Nagurney (1993).

Importantly, we have the following result, which is immediate from Dupuis and Nagurney (1993):

Theorem 2

The set of stationary points of the projected dynamical system (44) coincides with the set of solutions of the variational inequality problem (35) and is, thus, according to Definition 1, a *global supply chain network equilibrium. Hence, a vector* X^* *satisfying* $0 = \Pi_{\mathcal{K}}(X^*, -F(X^*))$ *also satisfies variational inequality (35).*

4. Qualitative Properties

In this Section, we provide some qualitative properties of the solution to variational inequality (35). In particular, we derive existence and uniqueness results. We also investigate properties of the function F (cf. (38)) that enters the variational inequality of interest here. Finally, we establish that the trajectories of the projected dynamical system (44) are well-defined under reasonable assumptions.

Since the feasible set is not compact we cannot derive existence simply from the assumption of continuity of the functions. Nevertheless, we can impose a rather weak condition to guarantee existence of a solution pattern. Let

$$\mathcal{K}_b = \{(Q^1, Q^2, Q^3, \gamma, \rho_3) | 0 \leq Q^1 \leq b_1;\ 0 \leq Q^2 \leq b_2;\ 0 \leq Q^3 \leq b_3;\ 0 \leq \gamma \leq b_4;\ 0 \leq \rho_3 \leq b_5\}, \tag{45}$$

where $b = (b_1, b_2, b_3, b_4, b_5) \geq 0$ and $Q^1 \leq b_1; Q^2 \leq b_2; Q^3 \leq b_3; \gamma \leq b_4; \rho_3 \leq b_5$ means that $q^{il}_{jhm} \leq b_1$; $q^{j}_{kh\hat{l}m} \leq b_2$; $q^{il}_{kh\hat{l}m} \leq b_3$; $\gamma_j \leq b_4$; and $\rho_{3khl} \leq b_5$ for all $i, l, j, k, h, \hat{l}, m$. Then $\mathcal{K}_b$ is a bounded closed convex subset of $R^{2ILJH+2JKHL+ILKHL+J+KHL}$. Thus, the following variational inequality

$$\langle F(X^b)^T, X - X^b \rangle \geq 0, \quad \forall X^b \in \mathcal{K}_b, \tag{46}$$

admits at least one solution $X^b \in \mathcal{K}_b$, from the standard theory of variational inequalities, since $\mathcal{K}_b$ is compact and F is continuous. Following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney (1999)), we then have:

Theorem 3

Variational inequality (35) admits a solution if and only if there exists a $b > 0$*, such that variational inequality (46) admits a solution in* $\mathcal{K}_b$ *with*

$$Q^{1b} < b_1, \quad Q^{2b} < b_2, \quad Q^{3b} < b_3, \quad \gamma^b < b_4, \quad \rho_3^b < b_5. \tag{47}$$

Theorem 4: Existence

Suppose that there exist positive constants M, N, R, with $R > 0$, such that:

$$\frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} + \frac{\partial c_j(Q^1)}{\partial q^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} + \omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}}$$

$$+\vartheta_j\frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}} + (\alpha^{il}+\beta_j)(\eta^{il}+\eta^{il}_{jm}) \geq M, \quad \forall Q^1 \text{ with } q^{il}_{jhm} \geq N, \forall i,l,j,h,m, \tag{48}$$

$$\frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} + \hat{c}^{il}_{kh\hat{l}}(Q^3) + \omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} + (\alpha^{il}+\delta_{kh\hat{l}})(\eta^{il}+\eta^{il}_{k\hat{l}}) \geq M,$$

$$\forall Q^3 \text{ with } q^{il}_{kh\hat{l}} \geq N, \quad \forall i,l,k,h,\hat{l}, \tag{49}$$

$$\frac{\partial c^j_{khl}(q^j_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} + \hat{c}^j_{kh\hat{l}m}(Q^2) + \vartheta_j\frac{\partial r^j(Q^1,Q^2)}{\partial q^j_{kh\hat{l}m}} + \delta_{kh\hat{l}}\eta^j_{k\hat{l}m} \geq M,$$

$$\forall Q^2 \text{ with } q^j_{kh\hat{l}m} \geq N, \quad \forall j,k,h,\hat{l},m, \tag{50}$$

$$d_{kh\hat{l}}(\rho_3) \leq N, \quad \forall \rho_3 \text{ with } \rho_{3kh\hat{l}} > R, \quad \forall k,h,\hat{l}. \tag{51}$$

Then variational inequality (35); equivalently, variational inequality (38), admits at least one solution.

Proof: Follows using analogous arguments as the proof of existence for Proposition 1 in Nagurney and Zhao (1993). □

Assumptions (48), (49), and (50) are reasonable from an economics perspective, since when the product transaction between a manufacturer in a country and a retailer or a manufacturer and a demand market in a country (and currency) or a retailer and demand market is large, we can expect the corresponding sum of the associated marginal costs of production, transaction, handling, and the weighted marginal risks and emissions generated to exceed a positive lower bound. Moreover, in the case where the demand price of the product in a currency and country at a demand market is high (cf. (51)), we can expect that the demand for the product at the demand market to not exceed a positive bound.

We now establish additional qualitative properties both of the function F that enters the variational inequality problem (cf. (35) and (38)), as well as uniqueness of the equilibrium pattern. Since the proofs of Theorems 5 and 6 below are similar to the analogous proofs in Nagurney and Ke (2001) they are omitted here. Additional background on the properties establish below can be found in the books by Nagurney (1999) and Nagurney and Dong (2002).

We first recall the concept of *additive production cost*, which was introduced by Zhang and Nagurney (1996) in the stability analysis of dynamic spatial oligopolies, and has also been utilized in the qualitative analysis of supply chain networks by Nagurney, Dong, and Zhang (2002).

Definition 2: Additive Production Cost

We term a production cost an additive production cost if for manufacturer il, the production cost f^{il} is of the following form:

$$f^{il}(q) = f^{il1}(q^{il}) + f^{il2}(\bar{q}^{il}), \tag{52}$$

where f^{il1} is the internal production cost that depends solely on the manufacturer's own output level and $f^{il2}(\bar{q}^{il})$ is the interdependent part of the production cost that is a function of all the other manufacturer's output levels $\bar{q}^{il} = (q^{11}, \dots, q^{il-1}, q^{il+1}, \dots, q^{il})$ and reflects the impact of the other manufacturers' production patterns on manufacturer il's cost.

Using the assumption of additive production costs, as well as several additional assumptions, we now state the following:

Theorem 5: Monotonicity

Suppose that the production cost functions f^{il}; $i = 1, \dots, I$, the risk functions r^{il}; $i = 1, \dots, I$; $l = 1, \dots, L$, and r^j; $j = 1, \dots, J$, are convex and that the c^{il}_{jhm}, $c^{il}_{kh\hat{l}}$, c_j, $\hat{c}^{il}_{jhm}$, and $c^{j}_{kh\hat{l}m}$ functions are convex; the $\hat{c}^{j}_{kh\hat{l}m}$ and $\hat{c}^{il}_{kh\hat{l}}$ functions are monotone increasing, and the $d_{kh\hat{l}}$ functions are monotone decreasing functions, for all $i, l, j, h, k, \hat{l}, m$. Assume also that the production cost functions are additive for all manufacturers il according to Definition 2. Then the vector function F that enters the variational inequality (38) is monotone, that is,

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}. \tag{53}$$

Monotonicity plays a role in the qualitative analysis of variational inequality problems similar to that played by convexity in the context of optimization problems. Under slightly stronger conditions, we obtain the following sharper result.

Theorem 6: Strict Monotonicity

Assume all the conditions of Theorem 5. In addition, suppose that one of the families of convex functions c^{il}_{jhm}*;* $i = 1, ..., I$*;* $l = 1, \ldots, L$*;* $j = 1, ..., J$*;* $h = 1, \ldots, H$*;* $m = 1, 2$*;* $c^{il}_{kh\hat{l}}$*;* $i = 1, ..., I$*;* $l = 1, \ldots, L$*;* $k = 1, ..., K$*;* $h = 1, \ldots, H$*;* $\hat{l} = 1, \ldots, L$*,* c_j*;* $j = 1, ..., J$*;* $\hat{c}^{il}_{jhm}$*;* $i = 1, \ldots, I$*;* $l = 1, \ldots, L$*;* $j = 1, \ldots, J$*;* $h = 1, \ldots, H$*;* $m = 1, 2$*, and* $c^{j}_{kh\hat{l}m}$*;* $j = 1, \ldots, J$*;* $k = 1, \ldots, K$*;* $h = 1, \ldots, H$*,* $\hat{l} = 1, \ldots, L$*;* $m = 1, 2$*, is a family of strictly convex functions. Suppose also that* $\hat{c}^{j}_{kh\hat{l}m}$*;* $j = 1, ..., J$*;* $k = 1, ..., K$*;* $h = 1, \ldots, H$*;* $\hat{l} = 1, \ldots, L$*;* $m = 1, 2$*;* $\hat{c}^{il}_{kh\hat{l}}$*;* $i = 1, ..., I$*;* $l = 1, \ldots, L$*;* $k = 1, ..., K$*;* $h = 1, \ldots, H$*;* $\hat{l} = 1, \ldots, L$ *and -*$d_{kh\hat{l}}$*,* $k = 1, ..., K$*,*$h = 1, \ldots, H$*;* $\hat{l} = 1, \ldots, \hat{L}$*, are strictly monotone. Then, the vector function* F *that enters the variational inequality (38) is strictly monotone, with respect to* $(Q^1, Q^2, Q^3, \gamma, \rho_3)$*, that is, for any two* X', X'' *with* $(Q^{1'}, Q^{2'}, Q^{3'}, \gamma^{'}, \rho_3') \neq (Q^{1''}, Q^{2''}, Q^{3''}, \gamma^{''}, \rho_3'')$

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle > 0. \tag{54}$$

Theorem 7: Uniqueness

Assuming the conditions of Theorem 6, there must be a unique equilibrium product pattern (Q^{1*}, Q^{2*}, Q^{3*})*, and a unique demand price price vector* ρ_3^* *satisfying the equilibrium conditions of the global supply chain network. In other words, if the variational inequality (35) admits a solution, then that is the only solution in* (Q^1, Q^2, Q^3, ρ_3)*.*

Proof: Under the strict monotonicity result of Theorem 6, uniqueness follows from the standard variational inequality theory (cf. Kinderlehrer and Stampacchia (1980)) □

Theorem 8: Lipschitz Continuity

The function that enters the variational inequality problem (37) is Lipschitz continuous, that is,

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \text{ where } L > 0, \tag{55}$$

under the following conditions:

(i). f^{il}, r^{il}, r^j, c^{il}_{jhm}, $c^{il}_{kh\hat{l}}$, c_j, $\hat{c}^{il}_{jhm}$, $c^j_{kh\hat{l}m}$ *have bounded second-order derivatives, for all* $i, l, \hat{l}, j, h, k, m$*;*

(ii). $\hat{c}^j_{kh\hat{l}m}$, $\hat{c}^{il}_{kh\hat{l}}$ *and* $d_{kh\hat{l}}$ *have bounded first-order derivatives for all* $j, k, h, l, \hat{l}, m$*.*

Proof: The result is direct by applying a mid-value theorem from calculus to the vector function F that enters the variational inequality problem (35). □

Theorem 9: Existence and Uniqueness

Assume the conditions of Theorem 8. Then, for any $X_0 \in \mathcal{K}$*, there exists a unique solution* $X_0(t)$ *to the initial value problem (44).*

Note that Theorem 9, unlike Theorems 4 and 7, is concerned with the existence of a unique trajectory. Theorems 4 and 7, on the other hand, are concerned with the existence and uniqueness of an equilibrium pattern. Hence, according to Theorem 9, the dusequilibrium dynamics of the global supply chain network are well-defined. Also, for completeness, we now provide a stability result (see Zhang and Nagurney (1996)). First we recall the following:

Definition 3: Stability of the System

The system defined by (44) is stable if, for every X_0 *and every equilibrium point* X^**, the Euclidean distance* $\|X^* - X_0(t)\|$ *is a monotone nonincreasing function of time* t*.*

We now provide a stability result.

Theorem 10: Stability of the Global Supply Chain Network

Assume the conditions of Theorem 5. Then the dynamical system (44) underlying the global supply chain network is stable.

Proof: Under the assumptions of Theorem 5, $F(X)$ is monotone and, hence, the conclusion follows directly from Theorem 4.1 of Zhang and Nagurney (1996). $\square$

In the next Section, we propose a discrete-time algorithm, the Euler method, which will track the dynamic trajectories until a stationary state is reached; equivalently, until an equilibrium point is reached satisfying Definition 1.

5. The Euler Method

In this Section, we consider the computation of a stationary of (44). The algorithm that we propose is the Euler-type method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). It has been applied to-date to solve a plethora of dynamic network models (see, e.g., Nagurney and Zhang (1996) and Nagurney and Dong (2002)). The algorithm not only provides a discretization of the continuous time trajectory defined by (44) but also yields a stationary, that is, an equilibrium point that satisfies variational inequality (35).

The Euler Method

Step 0: Initialization

Set $X^0 = (Q^{10}, Q^{20}, Q^{30}, \gamma^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter and set $\mathcal{T} = 1$. Set the sequence $\{a_{\mathcal{T}}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} a_{\mathcal{T}}$, $a_{\mathcal{T}} > 0$, $a_{\mathcal{T}} \to 0$, as $\mathcal{T} \to \infty$ (which is a requirement for convergence).

Step 1: Computation

Compute $X^{\mathcal{T}} = (Q^{1\mathcal{T}}, Q^{2\mathcal{T}}, Q^{3\mathcal{T}}, \gamma^{\mathcal{T}}, \rho_3^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + a_{\mathcal{T}} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{56}$$

Step 2: Convergence Verification

If $|X^{\mathcal{T}} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T}+1$, and go to Step 1.

Convergence results for the Euler method can be found in Dupuis and Nagurney (1993). See the book by Nagurney and Dong (2002) for applications of this algorithm to other supernetwork problems in the context of dynamic supply chains and financial networks with intermediation.

Variational inequality subproblem (56) can be solved explicitly and in closed form. This is due to the simplicity of the feasible set $\mathcal{K}$ as formulated above. For completeness, and also to illustrate the simplicity of the proposed computational procedure in the context of the global supply chain network model, we provide the explicit formulae for the computation of the $Q^{1\mathcal{T}}$, the $Q^{2\mathcal{T}}$, the $Q^{3\mathcal{T}}$, the $\gamma^{\mathcal{T}}$, and the $\rho_3^{\mathcal{T}}$ below.

Computation of the Product Transactions

In particular, compute, at iteration $\mathcal{T}$, the $q^{il\mathcal{T}}_{jhm}$s according to:

$$q^{il\mathcal{T}}_{jhm} = \max\{0, q^{il\mathcal{T}-1}_{jhm} - a_{\mathcal{T}}(\frac{\partial f^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il\mathcal{T}-1}_{jhm})}{\partial q^{il}_{jhm}} + \frac{\partial c_j(Q^{1\mathcal{T}-1})}{\partial q^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(q^{il\mathcal{T}-1}_{jhm})}{\partial q^{il}_{jhm}}$$

$$+\omega^{il}\frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q^{il}_{jhm}} + \vartheta_j \frac{\partial r^{j}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q^{il}_{jhm}} + (\alpha^{il} + \beta_j)(\eta^{il} + \eta^{il}_{jm}) - \gamma^{\mathcal{T}-1}_j)\}$$

$$\forall i, l, j, h, m; \tag{57}$$

the $q^{il\mathcal{T}}_{kh\hat{l}}$s according to:

$$q^{il\mathcal{T}}_{kh\hat{l}} = \max\{0, q^{il\mathcal{T}-1}_{kh\hat{l}} - a_{\mathcal{T}}(\frac{\partial f^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(q^{il\mathcal{T}-1}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} + \hat{c}^{il}_{kh\hat{l}}(Q^{3\mathcal{T}-1})$$

$$+\omega^{il}\frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q^{il}_{kh\hat{l}}} - (\alpha^{il} + \delta_{kh\hat{l}})(\eta^{il} + \eta^{il}_{k\hat{l}}) - \rho^{\mathcal{T}-1}_{3kh\hat{l}})\}, \qquad \forall i, l, k, h, \hat{l}, \tag{58}$$

and the $q^{j\mathcal{T}}_{kh\hat{l}m}$s, according to:

$$q^{j\mathcal{T}}_{kh\hat{l}m} = \max\{0, q^{j\mathcal{T}-1}_{kh\hat{l}m} - a_{\mathcal{T}}(\frac{\partial c^{j}_{kh\hat{l}m}(q^{j\mathcal{T}-1}_{kh\hat{l}m})}{\partial q^{j}_{kh\hat{l}m}} + \vartheta_j \frac{\partial r^{j}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q^{j}_{kh\hat{l}m}} + \hat{c}^{j}_{kh\hat{l}m}(Q^{2\mathcal{T}-1})$$

$$+\delta_{kh\hat{l}}\eta^{j}_{k\hat{l}m} + \gamma^{\mathcal{T}-1}_j - \rho^{\mathcal{T}-1}_{3kh\hat{l}})\}, \quad \forall j, k, h, \hat{l}, m. \tag{59}$$

Computation of the Prices

At iteration $\mathcal{T}$, compute the $\gamma^{\mathcal{T}}_j$s according to:

$$\gamma^{\mathcal{T}}_j = \max\{0, \gamma^{\mathcal{T}-1}_j - a_{\mathcal{T}}(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il\mathcal{T}-1}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q^{j\mathcal{T}-1}_{kh\hat{l}m})\}, \quad \forall j, \tag{60}$$

whereas the $\rho^{\mathcal{T}}_{3kh\hat{l}}$s are computed explicitly and in closed form according to:

$$\rho^{\mathcal{T}}_{3kh\hat{l}} = \max\{0, \rho^{\mathcal{T}-1}_{3kh\hat{l}} - a_{\mathcal{T}}(\sum_{j=1}^{J}\sum_{m=1}^{2} q^{j\mathcal{T}-1}_{kh\hat{l}m} + \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il}_{kh\hat{l}} - d_{kh\hat{l}}(\rho_3^{\mathcal{T}-1}))\}, \quad \forall k, h, \hat{l}. \tag{61}$$

Hence, at a given iteration, all the product transactions and prices can be solved explicitly and in closed form using the above simple formulae. Note that these computations can be done simultaneously, that is, in parallel. The algorithm also can be interpreted as a discrete-time adjustment process in which the product transactions between tiers adjust as well as the prices at the tiers until the equilibrium state is reached. Convergence conditions for the algorithm can be found in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996).

Note that one may recover the total emissions generated by a particular manufacturer in a country by simply computing the expression (14), with the product transactions at their equilibrium values, and with the summation over all manufacturers in all countries yielding the total number of emissions generated by all the manufacturers. The total amount of emissions generated by the consumers, in turn, in their transactions (cf. (31)) can be obtained by computing the expression: $\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\eta^{j}_{k\hat{l}m}\sum_{h=1}^{H} q^{j*}_{kh\hat{l}m}$.

6. Conclusions and Directions for Future Research

In this paper, we have proposed a framework for the formulation, analysis, and computation of solutions to global supply chain network problems with multicriteria decision-makers and environmental concerns in the presence of electronic commerce. In particular, we have proposed a global supply chain supernetwork consisting of three tiers of decision-makers: the manufacturers who are located in different countries and can trade in different currencies, the retailers, who can be either physical or virtual and need not be country specific, and the consumers associated with the demand markets in different countries who can transact in different currencies. We allowed for both physical and electronic transactions in the form of electronic commerce between manufacturers and retailers and between retailers and the consumers at the demand markets. Moreover, consumers can also obtain the products directly from the manufacturers through e-commerce. We presented both static and dynamic versions of the global supply chain network model with environmental decision-making and linked the equilibrium points of the former with the stationary points of the latter.

This framework generalizes the recent work of Nagurney and Toyasaki (2003) in supply chain supernetworks and environmental criteria to the global dimension and to include also explicit risk minimization, which is of a form sufficiently general to also capture environmental risks associated with the various transactions. Theoretical results were obtained along with a proposed discrete-time algorithm for the discretization of the continuous time product transaction and price trajectories. Finally, we demonstrated how the total amount of emissions generated can also be recovered from the equilibrium solution.

Future research may include the incorporation of a variety of policy instruments as well as applying the algorithm to concrete numerical examples.

References

Agrawal, V. and S. Seshadri (2000), "Risk Intermediation in Supply Chains," *IIE Transactions* **32**, 819-831.

Appelbaum, A. (2002a), "Europe Cracks Down on E-Waste," *IEEE Spectrum*, May, 46-51.

Appelbaum, A. (2002b), "Recycling in the United States: The Promised Landfill," *IEEE Spectrum*, May, 50.

Appelbaum, A. (2002c), "Japan: Serious Business," *IEEE Spectrum*, May, 51.

Batterman, S. A. and M. Amann (1991), "Targeted Acid Rain Strategies Including Uncertainty," *Jounral of Environmental Management* **32**, 57-72.

Bazaraa, M. S., Sherali, H. D. and C. M. Shetty (1993), **Nonlinear Programming: Theory and Algorithms**, John Wiley & Sons, New York.

Bloemhof-Ruwaard, J. M., Beek, P., Hordijk, L. and L. N. Van Wassenhove (1995), "Interactions between Operational Research and Environmental Management," *European Jounral of Operational Research* **85**, 229-243.

Buck de, A. J., Hendrix, E. M. T. and H.B. Schoorlemmer (1999), "Analysing Production and Environmental Risks in Arable Farming Systems: A Mathematical Approach," *European Journal of Operational Research* **119**, 416-426.

Chankong, V. and Y. Y. Haimes (1983), **Multiobjective Decision Making: Theory and Methodology**, North-Holland, New York.

Cohen, M. A. and A. Huchzermeier (1998), "Global Supply Chain Management: A Survey of Research and Applications," in **Quantitative Models for Supply Chain Management**, S. Tayur, M. Magazine, and R. Ganeshan, editors, Kluwer Academic Publishers, New York.

Cohen, M. A. and S. Mallik (1997), "Global Supply Chains: Research and Applications," *Production and Operations Management* **6**, 193-210.

Dhanda, K. K., Nagurney, A. and P. Ramanujam (1999), **Environmental Networks: A Framework for Economic Decision-Making and Policy Analysis**, Edward Elgar Publishers, Cheltenham, England.

Dupuis, P. and A. Nagurney (1993), "Dynamical Systems and Variational Inequalities," *Annals of Operations Research* **44**, 9-42.

Erenguc, S. S., Simpson, N. C. and A. J. Vakharia (1999), "Integrated Production/Distribution Planning in Supply Chains: An Invited Review," *European Journal of Operational Research* **115**, 219-236.

Fabian T. (2000), " Supply Chain Management in an Era of Social and Environment Accountability," *Sustainable Development International* **2**, 27-30.

Fishburn, P. C. (1970), **Utility Theory for Decision Making**, John Wiley & Sons, New York.

Gabay, D. and H. Moulin (1980), "On the Uniqueness and Stability of Nash Equilibria in Noncooperative Games," in: **Applied Stochastic Control of Econometrics and Management Science**, A. Bensoussan, P. Kleindorfer, and C. S. Tapiero, editors, North-Holland, Amsterdam, The Netherlands, pp. 271-294.

Hill, K. E. (1997), "Supply-Chain Dynamics, Environmental Issues, and Manufacturing Firms," *Environment and Planning A* **29**, 1257-1274.

Huchzermeier, A. and M. A. Cohen (1996), "Valuing Operational Flexibility Under Exchange Rate Uncertainty," *Operations Research* **44**, 100-113.

Ingram, M. (2002), "Producing the Natural Fiber Naturally: Technological Change and the US Organic Cotton Industry," *Agriculture and Human Values* **19**, 325-336.

Johnson, M. E. (2001), "Learning from Toys: Lessons in Managing Supply Chain Risk from the Toy Industry," *California Management Review* **43**, 106-124.

Keeney, R. L. and H. Raiffa (1993), **Decisions with Multiple Objectives: Preferences**

and Value Tradeoffs, Cambridge University Press, Cambridge, England.

Kinderlehrer, D. and G. Stampacchia (1980), **An Introduction to Variational Inequalities and Their Application**, Academic Press, New York.

Kogut, B. and N. Kulatilaka (1994), "Options Thinking and Platform Investments: Investing in Opportunity," *California Management Review* **36**, 52-71.

Mullany, T. J., Green, H., Arndt, M., Hof, R. D. and L. Himelstein (2003), "The E-Biz Surprise," *Business Week Online*; see: `http://www.businessweek.com`

Nagurney, A. (1999), **Network Economics: A Variational Inequality Approach**, second and revised edition, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., Cruz, J. and D. Matsypura (2003), "Dynamics of Global Supply Chain Supernetworks," *Mathematical and Computer Modelling* **37**, 963-983.

Nagurney, A. and J. Dong (2002), **Supernetworks: Decision-Making for the Information Age**, Edward Elgar Publishers, Cheltenham, England.

Nagurney, A., Dong, J. and D. Zhang (2002), "A Supply Chain Network Equilibrium Model," *Transportation Research E* **38**, 281-303.

Nagurney, A. and K. Ke (2001), "Financial Networks with Intermediation," *Quantitative Finance* **1**, 441-451.

Nagurney, A., Loo, J., Dong, J. and D. Zhang (2002), "Supply Chain Networks and Electronic Commerce: A Theoretical Perspective," *Netnomics* **4**, 187-220.

Nagurney, A. and F. Toyasaki (2003), "Supply Chain Supernetworks and Environmental Criteria," *Transportation Research D* **8**, 185-213.

Nagurney, A. and F. Toyasaki (2005), "Reverse Supply Chain Management and Electronic Waste Recycling: A Multitiered Network Equilibrium Framework for E-Cycling," *Transportation Research E* **41**, 1-28.

Nagurney, A. and D. Zhang (1996), **Projected Dynamical Systems and Variational Inequalities with Applications**, Kluwer Academic Publishers, Boston, Massachusetts.

Nagurney, A. and L. Zhao (1993), "Networks and Variational Inequalities in the Formulation and Computation of Market Disequilibria: The Case of Direct Demand Functions," *Transportation Science* **27**, 4-15.

Nash, J. F. (1950), "Equilibrium Points in N-Person Games," in: *Proceedings of the National Academy of Sciences, USA* **36**, 48-49.

Nash, J. F. (1951), "Noncooperative Games," *Annals of Mathematics* **54**, 286-298.

Qio, Z., Prato, T. and F. McCamley (2001), "Evaluating Environmental Risks Using Safety-First Constraints," *American Journal of Agricultural Economics* **83**, 402-413.

Quinn, B. (1999), "E-System Manages Inventory to Reduce Risk," *Pollution Engineering* **31**, 27-28.

Smeltzer, L. R. and S. P. Siferd (1998), "Proactive Supply Management: The Management of Risk," *International Journal of Purchasing and Materials Management*, Winter, 38-45.

Yu, P. L. (1985), **Multiple Criteria Decision Making – Concepts, Techniques, and Extensions**, Plenum Press, New York.

Zhang, D. and A. Nagurney (1996), "Stability Analysis of an Adjustment Process for Oligopolistic Market Equilibrium Modeled as a Projected Dynamical System," *Optimization* **36**, 263-285.

Zsidisin, G. A. (2003), "Managerial Perceptions of Supply Risk," *The Journal of Supply Chain Management: A Global Review of Purchasing and Supply*, Winter, 14-25.